On the first day of Christmas my true love gave to me ...

By Byron Spice
Pittsburgh Post-Gazette

With Christmas just a few days off, the endless replays of "The Twelve Days of Christmas" are no doubt a-ringing in your ears. You're a-tiring of the numerous parodies and, along with some economists, may even have been a-tallying up the costs of all those gifts.

But have you stopped to think about the carol's mathematical aspects?

Let's face it, a lot of numbers are involved in this song. All of those swans a-swimming, pipers piping and lords a-leaping add up. The numbers and the way they add up make for some interesting patterns that, coincidentally or not, resemble such seasonal icons as Christmas trees, stockings and stars.

Analyzing "Twelve Days" is something of an old chestnut among some math educators, who use the song to discuss mathematical principles and history, said Mike Breen, a spokesman for the American Mathematical Society.

But don't look for hidden meaning in the song's numerology.

"We're not cracking the DaVinci Code," said Bill Butterworth, director of the mathematics program at the Barat College of DePaul University in Lake Forest, Ill.

The singer receives the gift of a partridge in a pear tree on one day, two turtle doves and a partridge in a pear tree on the second, three French hens, two turtle doves... at any rate, it amounts to a lot of stuff by the twelfth day.

How many gifts? Simple addition

The easiest, if longest, method to calculate the total is to simply add them up: the first day's gift (1), added to the second day's gifts (1 + 2), added to the third day's gifts (1 + 2 + 3) and so on. You end up with a total of 364 gifts — almost one gift for every day in the year.

Day of song	Number of gifts
One	1 +
Two	(1 + 2) +
Three	(1 + 2 + 3) +
Four	(1 + 2 + 3 + 4) +
Five	(1 + 2 + 3 + 4 + 5) +
Six	(1 + 2 + 3 + 4 + 5 + 6) +
Seven	(1 + 2 + 3 + 4 + 5 + 6 + 7) +
Eight	(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) +
Nine	(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) +
Ten	(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) +
Eleven	(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11) +
Twelve	(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)=
Total	**364**

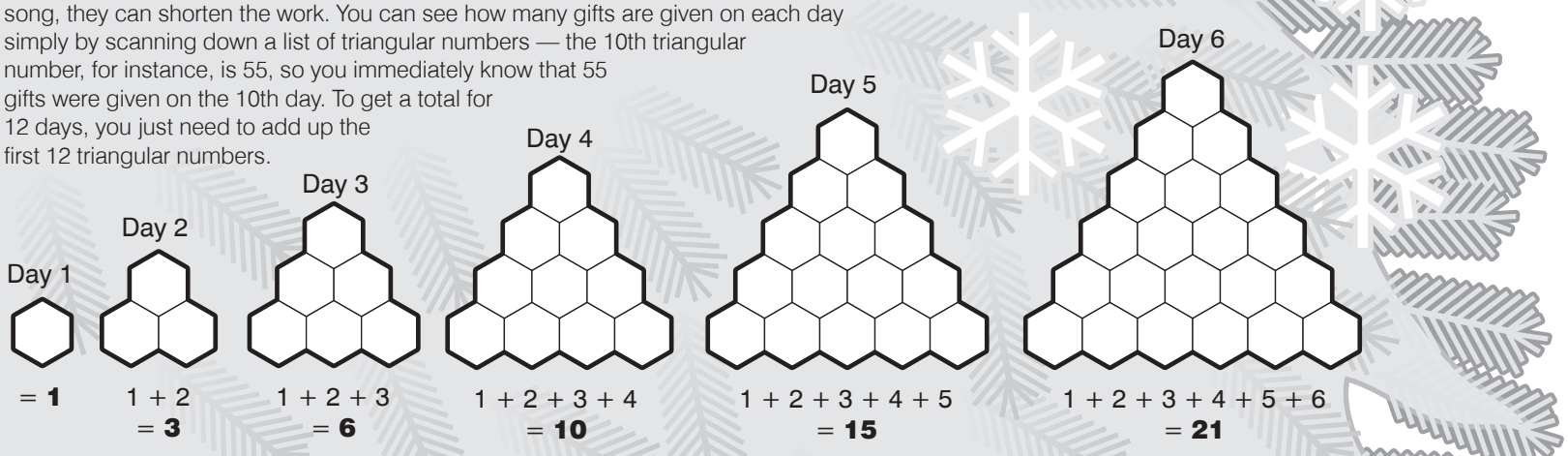
How much would it cost?

PNC Advisors calculates it would cost $65,264.28 to give your true love everything enumerated in the song.

Triangular numbers

The totals for each day — 1, 3, 6, 10, etc. — are what the ancient Greeks called "triangular numbers," because they can be arranged in a compact triangular pattern. During the holidays, you might call them "Christmas tree numbers."

Triangular numbers have a number of interesting properties. In the case of calculating gifts in the song, they can shorten the work. You can see how many gifts are given on each day simply by scanning down a list of triangular numbers — the 10th triangular number, for instance, is 55, so you immediately know that 55 gifts were given on the 10th day. To get a total for 12 days, you just need to add up the first 12 triangular numbers.



Pascal's Triangle

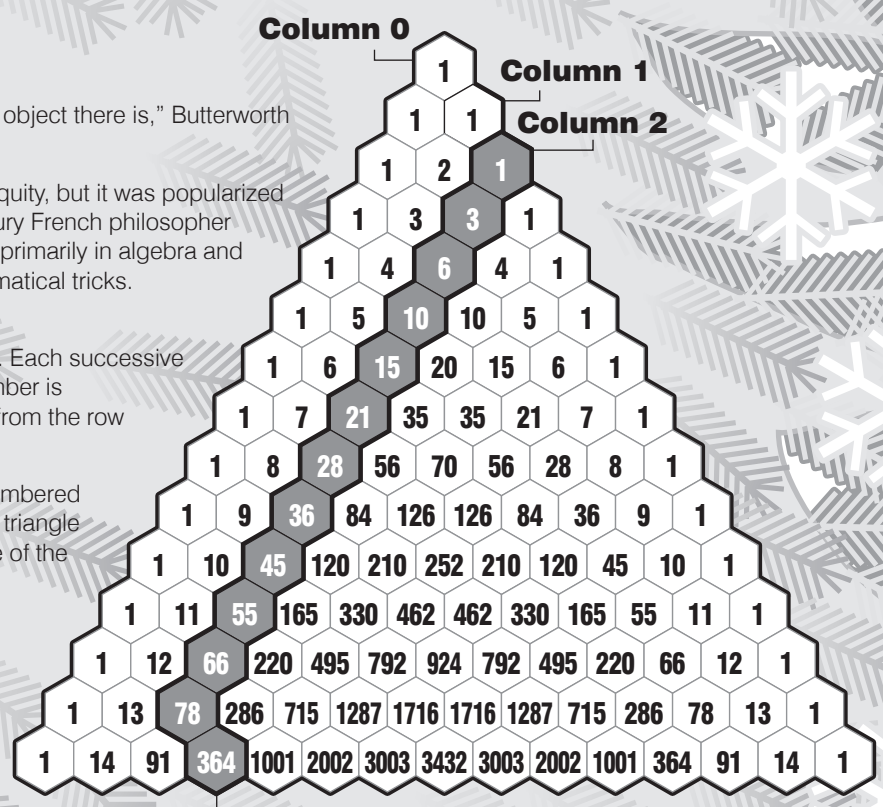
relies on a Christmas tree-like pattern.

"It's probably the single most important mathematical object there is," Butterworth said. "It's certainly the most accessible."

The origins of this arithmetical triangle go back to antiquity, but it was popularized and ultimately named after Blaise Pascal, a 17th-century French philosopher and mathematician. The triangle has numerous uses, primarily in algebra and in probabilities, and includes a number of odd mathematical tricks.

How it's constructed

You begin with the number 1 on the top of the triangle. Each successive row includes an additional number and each new number is calculated by adding the numbers on either side of it from the row above. It looks like this:



Hockey stick, or Christmas stocking, shape

By convention, the rows and diagonal columns are numbered beginning with zero, so the number 1 at the top of the triangle is row 0 and the diagonal columns of 1s on either side of the triangle are column 0.

If you sight down column two, you'll see the triangular numbers — 1, 3, 6, 10, 15, etc. And, thanks to what is called the Hockey Stick Theorem — or, more appropriately for the holidays, the Christmas Stocking Theorem — Pascal's Triangle makes it even easier to calculate the gift total.

The theorem works the same for any column: Move your finger down any column and stop at any number you choose. Now move your finger to the next row of numbers and move one column toward the center of the triangle. That number will be the sum of the numbers in your original column. If you highlight the original column and the sum, the pattern looks something like a stocking or a hockey stick.

In the case of "The Twelve Days of Christmas," move down column two — the column of triangular numbers — until you reach the 12th number, 78. Move your finger down one row and to the right and you find the total: **364.**

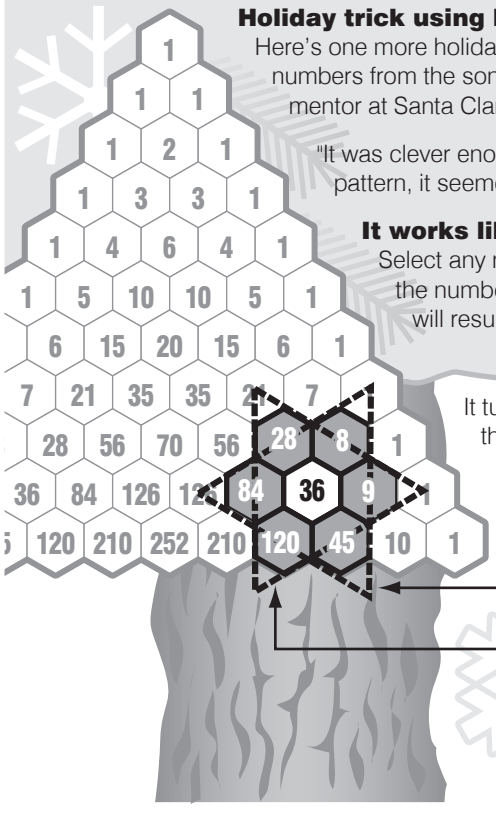
Holiday trick using Pascal's Triangle

Here's one more holiday-related trick from Pascal's Triangle, though it doesn't relate directly to the numbers from the song. The "Star of David Theorem" is something Butterworth learned from his mentor at Santa Clara University, the late David Logothetti.

"It was clever enough that I remembered it," Butterworth said and, because it results in a star pattern, it seemed to fit in with the theme of Christmas.

It works like this

Select any number inside the triangle. Identify the six numbers surrounding it. Now divide the numbers into two groups of three, so that a triangle drawn around each group of three will result in a Star of David.



It turns out that when you multiply the three numbers in either triangle together, the product equals the product of the three numbers in the other triangle. In this case,

45 X 8 X 84 = 30,240

28 X 9 X 120 = 30,240

For more information

Inside Science News Service:
www.insidescience.org/reports/2002/058.html

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Sources: American Institute of Physics, American Mathematical Society